Partieelbreuken: oefening

$$\int \frac{dx}{x^8 + x^6} = \int \frac{dx}{x^6(x^2+1)}$$: splitsen in partieelbreuken

$$\frac{1}{x^6(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x^5} + \frac{F}{x^6} + \frac{Gx+H}{x^2+1}$$

$$1 = Ax^7 + Ax^5 + Bx^6 + Bx^4 + Cx^5 + Cx^3 + Dx^4 + Dx^2 + Ex^3 + Ex$$
$$+ Fx^2 + F + Gx^7 + Hx^6$$

$$1 = F + Ex + (D+F)x^2 + (C+E)x^3 + (B+D)x^4 + (A+C)x^5$$
$$+ (B+H)x^6 + (A+G)x^7$$

Dan: $F = 1$ $E = 0$ $D = -1$ $C = 0$ $B = 1$ $A = 0$ $H = -1$ $G = 0$

$$\int \frac{dx}{x^8 + x^6} = \int \frac{dx}{x^2} - \int \frac{dx}{x^4} + \int \frac{dx}{x^6} - \int \frac{dx}{x^2+1} = -\frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} - Bg\tan x + C$$

Integralen

$$\int \frac{x\,dx}{1+x^2} = \frac{1}{2}\int \frac{dt}{t}$$

Substitutie: $\begin{cases} 1+x^2 = t \\ 2x\,dx = dt \end{cases}$

$$= \frac{1}{2}\ln t + C = \frac{1}{2}\ln(1+x^2) + C$$

$$\int \frac{\sqrt{\ln y}}{y}dy = \int \sqrt{t}\,dt$$

Substitutie: $\begin{cases} \ln y = t \\ \dfrac{dy}{y} = dt \end{cases}$

$$= \frac{2}{3}t^{3/2} + C = \frac{2}{3}\sqrt{(\ln y)^3} + C$$

$$\int \cos^3 x \sin x\,dx = -\int t^3\,dt$$

Substitutie: $\begin{cases} \cos x = t \\ -\sin x\,dx = dt \end{cases}$

$$= -\frac{t^4}{4} + C = -\frac{1}{4}\cos^4 x + C$$

Integralen

$$\int \frac{(a^x - b^x)^2 dx}{a^x b^x} = \int \frac{a^{2x} - 2a^x b^x + b^{2x}}{a^x b^x} dx = \int \frac{a^x}{b^x} dx - 2\int dx + \int \frac{b^x}{a^x} dx$$

Substituties: $\begin{cases} u = \left(\frac{a}{b}\right)^x \\ du = \left(\frac{a}{b}\right)^x \ln\left(\frac{a}{b}\right) dx \end{cases}$ $\begin{cases} v = \left(\frac{b}{a}\right)^x \\ dv = \left(\frac{b}{a}\right)^x \ln\left(\frac{b}{a}\right) dx \end{cases}$

$$= \int \frac{du}{\ln \frac{a}{b}} - 2x + \int \frac{du}{\ln \frac{b}{a}} = \frac{u}{\ln \frac{a}{b}} - 2x + \frac{u}{\ln \frac{b}{a}}$$

$$= \left(\frac{a}{b}\right)^x \frac{1}{\ln a - \ln b} - 2x + \left(\frac{b}{a}\right)^x \frac{1}{\ln b - \ln a} + C$$

$$= \left[\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x\right] \frac{1}{\ln a - \ln b} - 2x + C$$

Integralen

$$\int \frac{x}{\sqrt{1+4x}}dx = \frac{1}{16}\int \frac{(t-1)}{\sqrt{t}}dt$$

Substitutie: $\begin{cases} 1+4x=t \\ dx = dt/4 \end{cases}$

$$= \frac{1}{16}\int \sqrt{t}dt - \frac{1}{16}\int \frac{dt}{\sqrt{t}} = \frac{1}{24}(1+4x)^{3/2} - \frac{1}{8}(1+4x)^{1/2} + C$$

$$\int \sin^3 \varphi d\varphi = \int \sin^2 \varphi \sin \varphi \, d\varphi$$

Substitutie: $\begin{cases} \cos\varphi = t \\ d\cos\varphi = dt \end{cases}$

$$= -\int (1-\cos^2\varphi) d\cos\varphi$$

$$= -\int dt + \int t^2 dt$$

$$= -\cos\varphi + \frac{1}{3}\cos^3\varphi + C$$

Integralen (Oef. PI)

$$\int \ln(x^2+2)dx = x\ln(x^2+2) - \int x \frac{2x}{x^2+2}dx = x\ln(x^2+2) - 2\int \frac{x^2}{x^2+2}dx$$

$$= x\ln(x^2+2) - 2\int \frac{x^2+2-2}{x^2+2}dx$$

$$= x\ln(x^2+2) - 2x + 4\int \frac{1}{x^2+2}dx$$

$$= x\ln(x^2+2) - 2x + \frac{4}{2}\int \frac{1}{\left(x/\sqrt{2}\right)^2+1}dx$$

Substitutie:

$$\begin{cases} x/\sqrt{2} = t \\ dx = \sqrt{2}dt \end{cases}$$

$$= x\ln(x^2+2) - 2x + 2\sqrt{2}\int \frac{dt}{1+t^2}$$

$$= x\ln(x^2+2) - 2x + 2\sqrt{2}Bg\tan\frac{x}{\sqrt{2}} + C$$

Integralen (Oef. PB) (1/2)

$$\int \frac{2x^2-3x-3}{(x-1)(x^2-2x+5)}dx = ?$$

<u>Splitsen in partieelbreuken:</u> $$\frac{2x^2-3x-3}{(x-1)(x^2-2x+5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2-2x+5}$$

$$2x^2-3x-3 = Ax^2-2Ax+5A+Bx^2-Bx+Cx-C$$
$$= (A-B)x^2+(-2A-B+C)x+5A-C$$

Hieruit volgt: $$\begin{cases} A+B=2 \\ -2A-B+C=-3 \\ 5A-C=-3 \end{cases} \qquad \begin{cases} A=-1 \\ B=3 \\ C=-2 \end{cases}$$

$$\int \frac{2x^2-3x-3}{(x-1)(x^2-2x+5)}dx = -\int \frac{dx}{x-1} + \int \frac{3x-2}{x^2-2x+5}dx$$

Integralen (Oef. PB) (2/2)

$$\int \frac{2x^2-3x-3}{(x-1)(x^2-2x+5)}dx = -\int \frac{dx}{x-1} + \int \frac{3x-2}{x^2-2x+5}dx$$

$$= -\int \frac{dx}{x-1} + \frac{3}{2}\int \frac{2x-2+2/3}{x^2-2x+5}dx$$

$$= -\int \frac{dx}{x-1} + \frac{3}{2}\int \frac{2x-2}{x^2-2x+5}dx + \int \frac{1}{x^2-2x+5}dx$$

$$= -\ln(x-1) + \frac{3}{2}\int \frac{dt}{t} + \int \frac{1}{(x-1)^2+4}dx$$

$$= -\ln(x-1) + \frac{3}{2}\ln(x^2-2x+5) + \frac{1}{4}\int \frac{1}{((x-1)/2)^2+1}dx$$

$$= -\ln(x-1) + \frac{3}{2}\ln(x^2-2x+5) + \frac{1}{2}Bg\tan\left(\frac{x-1}{2}\right) + C$$

Substitutie:

$$\begin{cases} x^2-2x+5 = t \\ dx = (2x-2)^{-1}dt \end{cases}$$

Substitutie:

$$\begin{cases} (x-1)/2 = u \\ dx/2 = du \end{cases}$$