Harmonische trillingen

1) $mg = kx$

$a_y = \frac{-k}{m} y$

$\omega^2 = \frac{k}{m}$

$T = 2\pi \sqrt{\frac{m}{k}}$

$v_y = A\omega \cos(\omega t + \varphi_0)$

$x_y = A \sin(\omega t + \varphi_0)$

↳ zie cursus

2) $f = \frac{1}{T}$

↗ ↘ kleine waarde

$k >>$ (stugge veer)

↳ $m <<$ moet veel groot zijn

3) a) A: $mg = kx$

$1 = k \cdot 10 \cdot 10^{-2}$

$\boxed{k = 10}\ N/m$

b) $\frac{1}{f} = T = 2\pi \sqrt{\frac{m}{k}}$

$\Leftrightarrow f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

$= \frac{1}{2\pi} \sqrt{\frac{10}{0,2}}$

$= 1,13\ Hz$ ✓

c) $E_M = \frac{1}{2} k A^2$

k is gelijk aan beide veren,

dus A ook

$\Rightarrow E_{MA} = E_{MB}$ ✓

4) $a_y = \omega^2 A \sin(\omega t + \varphi_0 + \pi)$

$\vec{a_y} = -\omega^2 \vec{\lambda_y}$

$\rightarrow \vec{\lambda_y} = 0$

$\Rightarrow a_y = 0$

uitwijking = 0

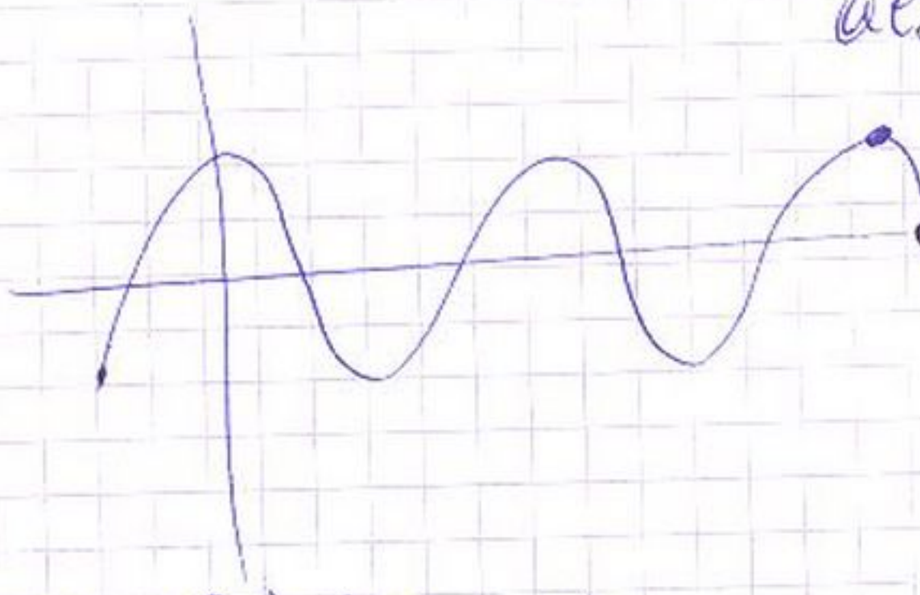
→ versnelling klein

dus versnelling groot

als uitwijking groot is

versnelling groot

versnelling klein



5) $m_1 = 0{,}3\ kg \quad x_1$

$m_2 = 0{,}5\ kg \quad x_2 = x_1 + 0{,}12\ m$

$m_3 = 1{,}0 kg \quad x_3$

$mg = kx \rightarrow \begin{cases} 3 = k \cdot x_1 \\ 5 = k(x_1 + 0{,}12\ m) \\ 10 = k \cdot x_3 \end{cases}$

~~$2 = k\left(\frac{3}{k} + 0{,}12\right)$~~ $5 = k\left(\frac{3}{k} + 0{,}12\right)$

~~$2 = 3 + 0{,}12k$~~ $5 = 3 + 0{,}12k$

~~$-1 = 0{,}12k$~~ $2 = 0{,}12k$

~~$k = -25/3$~~ $k = 50/3\ N/m$

$T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{1 \cdot 3}{50}}$

$= 1{,}545$

6) (k) $\Delta y,1 = \Delta y,2$

$\Leftrightarrow$ ~~2~~ $\sin\left(\pi t + \frac{\pi}{6}\right) = 2\sin\left(\pi t + \frac{\pi}{2}\right)$

$\Leftrightarrow \sin\left(\pi t + \frac{\pi}{6}\right) - \sin\left(\pi t + \frac{\pi}{2}\right) = 0$

$\Leftrightarrow 2\sin \frac{\pi t + \pi/6 - \pi t - \pi/2}{2} \cdot$

$\cos \frac{\pi t + \pi t + \pi/6 + \pi/2}{2} = 0$

$\Leftrightarrow \cos \frac{2\pi t + 4\pi/6}{2} = 0$

$\Leftrightarrow \cos\left(\pi t + \pi/3\right) = 0$

$\Leftrightarrow \pi t + \frac{\pi}{3} = \frac{\pi}{2} + k\pi$

$\Leftrightarrow t + \frac{1}{3} = \frac{1}{2} + k$

$\Rightarrow t = \left(\frac{1}{6} + k\right) s$ ✓

$\Delta y_1 = 2\sin\left(\pi t + \frac{\pi}{6}\right) = 2\sin\left(\pi t + \frac{\pi}{2}\right) = \Delta y_2$

$= 1{,}73\ m$

(k) maximale afstand:

~~amplitude van verschilfunctie~~

~~$\frac{1}{6} + k$~~

~~$\Delta y,1 = \quad = A\sin(\ldots)$~~

~~$= 2{,}0\ m$~~ $A = 2{,}0\ m$ ✓

(k) fasehoek

$\cos\left(\pi t + \frac{\pi}{3}\right) = 0$ → samengestelde cirkel

$\varphi = \frac{\pi}{3} = 60°$

7) $m = 0,200$ kg

$A = 1,0$ m

$\omega = \pi/5$

$\varphi_0 = \pi/3$

Gevr/ $E_m = ?$

$a = ?$ (bij $A_y = 1,0$ m)

$\omega^2 = \frac{k}{m}$

$\Rightarrow k = \omega^2 m = 0,2\pi^2$

$E_m = \frac{1}{2} k A^2$

$= \frac{1}{2} 0,2\ \pi^2\ 1^2$

$= 0,1\ \pi^2$

$= 0,99$ J ✓

$a_y = \omega^2 u_y$

$= \pi^2 . 1$

$= 9,9$ m/s² ✓

8) $m = 0,50$ kg

$k = 25$ N/m

$u_{y\,max} = 2A$ ⇒ $v_{y,max} = A\ \omega$

Sorry bij opgave 9

$u_y = A \rightarrow E_m = ?$

$A = \frac{9,5}{44}$ m $= 0,012$ m

~~$E_m = \frac{1}{2} k (2A)^2$~~ ~~$E_{m_1} = E_{m_2}$~~

$mg = kx$

met $x = A$ in evenwichtstand

~~$= \frac{25}{2} \cdot 4A^2$~~ ~~$\frac{1}{2} k A^2 = \frac{1}{2} k 4A^2$~~

~~$= 50A^2$~~ ?

$\omega^2 = \frac{k}{m} = 25 . 2 \Rightarrow \omega = \sqrt{50}$

$A = 0,20$ m $\quad u_y = 2A = A \sin(\omega t + \varphi)$

$\Rightarrow 2A = A \sin(\sqrt{50}\, t + \varphi)$

$E_M = \frac{1}{2} k A^2$

9) $m = 50.10^{-3}$ kg

$T = 0,50$ s $\rightarrow$ t ap $v_{max} = 0,125$ s

$v_{max} = 15$ cm/s = ~~15~~ 15.10^{-2} m/s

$\begin{cases} A = ? \\ k = ? \\ a_{max} = ? \end{cases}$

$\omega = \frac{2\pi}{T} = 12,6$ rad/s

$\omega^2 = \frac{k}{m} \Leftrightarrow k = \omega^2 m = 158 . 50.10^{-3}$

$\boxed{k = 7,9 \text{ N/m}}$ ✓

$v_{max} = A\omega \sin(\omega t)$

$\Leftrightarrow 0,15 = A . 12,6 . \sin(12,6 . 0,125)$

$\Leftrightarrow \boxed{A = 0,012 \text{ m}}$ ✓

$a_y = \omega . v_y$

$= \boxed{1,89 \text{ m/s}^2}$ ✓

10) $m = 3,0$ kg ; EHT

$y = 5,0 \cos\left(\frac{\pi}{3} t - \frac{\pi}{4}\right)$

$= 5,0 \sin\left(\frac{\pi}{3} t - \frac{\pi}{4} + 2\frac{\pi}{4}\right)$

$= 5,0 \sin\left(\frac{\pi}{3} t + \frac{\pi}{4}\right)$

$A = 5,0$ m

$\omega = \frac{\pi}{3}$ rad/s

$\Delta t = ?$

$\varphi_0 = \frac{\pi}{4}$

$E_{p,e} = \frac{1}{2} E_M$

$\Leftrightarrow \frac{1}{2} k y^2 = \frac{1}{2}\left(\frac{1}{2} k A^2\right)$

$\Leftrightarrow y = \frac{A}{\sqrt{2}} = 3,54$ m ✓

14) $m = 0,100\ kg$

$\lambda_y = 0,392\ m$

$A = 0,150\ m$

$T = ?$

$v_{max} = ?$

$mg = kx$

$0,100 \cdot 10 = k \cdot 0,392$

$k = 2,55\ N/m$

$\omega^2 = \frac{k}{m} = \sqrt{\frac{2,55}{0,100}}$

$\omega = 5,05\ rad/s$

$T = \frac{2\pi}{\omega} = 1,245$ ✓

$v_{max} = \omega \cdot A$

$= 0,758\ m/s$ ✓

16) $\frac{\lambda_y}{A} = ?$

$E_k = 3\,E_{pe}$

~~$\frac{1}{2} k A^2 \cos^2(\omega t + \varphi_0) = 3 \cdot \frac{1}{2} k A^2 \cos^2(\omega t + \varphi_0)$~~

~~$A^2 \cos^2(\omega t + \varphi_0) = 3 \lambda_y^2$~~

$E_M - E_{pe} = 3\,E_{pe}$

$\frac{1}{2} k A^2 - \frac{1}{2} k \lambda_y^2 = 3 \cdot \frac{1}{2} k \lambda_y^2$

$A^2 - \lambda_y^2 = 3 \lambda_y^2$

$A^2 = 4 \lambda_y^2$

$\frac{\lambda_y}{A} = \sqrt{\frac{1}{4}} = 0,5$ ✓

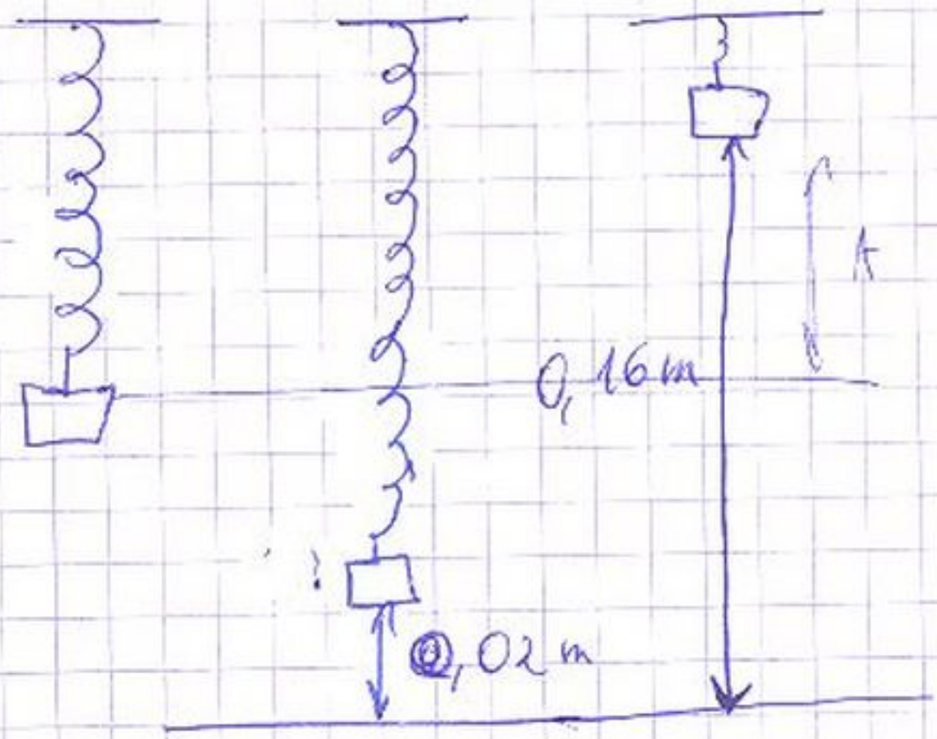
19) $m = 0{,}300\ kg$

$T = 4{,}0\ s$

$\omega = \frac{2\pi}{T} = 1{,}57\ rad/s$

$\omega^2 m = k$

$\boxed{k = 0{,}74\ N/m}$ ✓



b) $v = ?$

$a = ?$

$u_y = 0 \rightarrow$ evenwichtsstand

$\Leftrightarrow u_y = A \sin(\omega t + \varphi_0)$

$\varphi_0 = 0$

en bereik van $= 0{,}14\ m \rightarrow A = 0{,}07\ m$

$v_y = A\omega \cos(\omega t + \varphi_0)$

$= 0{,}07 \cdot 1{,}57 \cdot 1$

$= 0{,}11\ m/s$ ✓

$a_y = \omega^2 \cdot u_y$

$= 0\ m/s^2$ ✓

22) $m = 0{,}100\ kg$

$t = 5{,}0\ s$

$\rightarrow$ 5 periodes afgelegd

$\rightarrow T = 1{,}0\ s$

$f = \frac{1}{T} = 1{,}0\ Hz$ ✓

23) $m = 5,22\ kg$

$v_{max} = 15,3\ cm/s = 0,153\ m/s$

$T = 645\ ms = 645 \cdot 10^{-3}\ s$ → $t = 161,25$

$k = ?$ $A = ?$

$\omega = \frac{2\pi}{T} = 9,74\ rad/s$

$\omega^2 = \frac{k}{m}$

$k = \omega^2 \cdot m$

$= \boxed{495\ N/m}$ ✓

$v_y = A\omega \cos(\omega \cdot t)$

$0,153 = A \cdot 9,74 \cdot \cos(9,74 \cdot 161,25)$

$A = \boxed{0,016\ m}$ ✓

24) $k = 456\ N/m$

t, $v_y = 0$

↳ $x = 0,112\ m$, $v_x = -13,6\ m/s$, $a_x = -123\ m/s^2$

$m = ?$, $f = ?$, $A = ?$

$mg = k\,x$ want evenwichtsstand

$m = \frac{456 \cdot 0,112}{10} = 5,1\ kg$

$\omega = \sqrt{\frac{k}{m}} = 9,5\ rad/s$

$f = \frac{\omega}{2\pi} = 1,5\ Hz$

$A = ?$

gebruik formule bewegingsvergelijking (1)

$v_y^2 = \omega^2 (A^2 - y^2)$

Vergelijkingen:

$\begin{cases} 0,112 = A \cos \omega t \\ -13,6 = A\omega \sin \omega t \\ -123 = -\omega^2 \cdot 0,112 \end{cases}$

↳ $\omega =$ ...

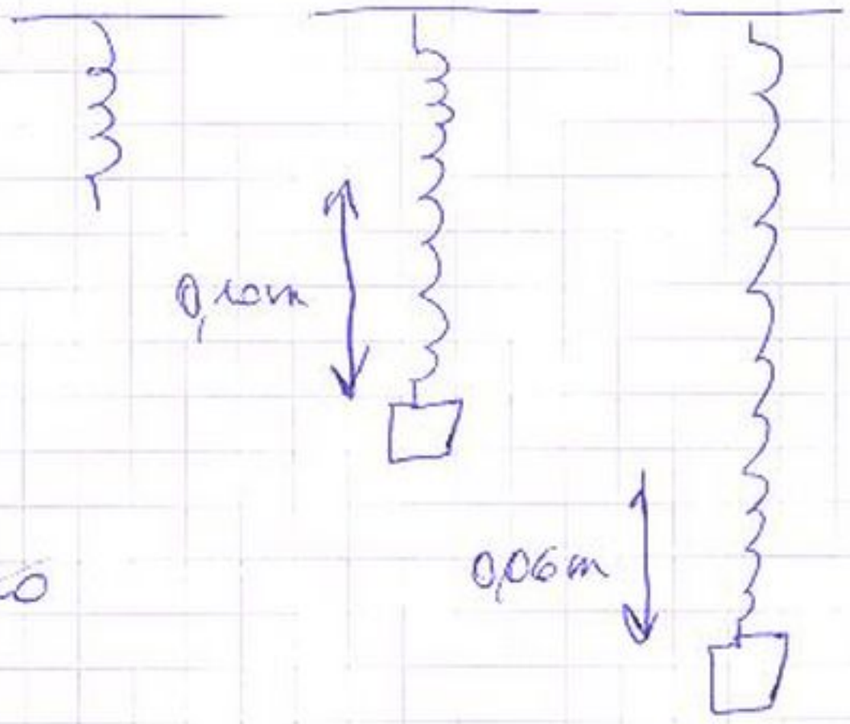
$T =$ ... $f =$ ...

25) $m = 0{,}100\ kg$

$y = A \sin(\omega t + \varphi_0)$

$A = 0{,}06\ m$

$\varphi_0 = \frac{\pi}{2}$



$mg = kx$

$0{,}100 \cdot 10 = k \cdot 0{,}10$

$k = 10\ N/m$

$\omega = \sqrt{\frac{k}{m}} = 10\ rad/s$

$\Rightarrow y = 0{,}06 \sin\left(10t + \frac{\pi}{2}\right)$

$= 0{,}06 \cos(10t)$ → o Vergeten op elev.!

26) $T = 2\pi\sqrt{\frac{l}{g}} \Leftrightarrow \sqrt{g} = \frac{2\pi}{T}\sqrt{l}$ moet je nog aan passen

slinger 1: $\sqrt{g} = \frac{2\pi}{T_1}\sqrt{l_1}$

slinger 2: $\sqrt{g} = \frac{2\pi}{T_2}\sqrt{l_2}$

$\frac{2\pi}{T_1}\sqrt{l_1} = \frac{2\pi}{T_2}\sqrt{l_2}$ met $12\,T_1 = 11\,T_2$ en $l_1 = 1\ m$

$\frac{\sqrt{1}}{\frac{11}{12}T_2} = \frac{\sqrt{l_2}}{T_2}$ ...

$l_2 = \left(\frac{12}{11}\right)^2 = 1{,}19\ m$